MATH 121A Prep: Bases

Facts to Know:

Let $\vec{v_1}, \ldots, \vec{v_n}$ be vectors in $\mathbb{R}^n$.

$\vec{v_1}, \ldots, \vec{v_n}$ are Linearly Independent if:

$\vec{v_1}, \ldots, \vec{v_n}$ Spans $\mathbb{R}^n$ if:

$\vec{v_1}, \ldots, \vec{v_n}$ is a Basis if:

Dimension of a Basis:

Examples:

1. Determine whether the vectors $\vec{v_1} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$, $\vec{v_2} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$, $\vec{v_3} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$ are linearly independent.

2. Show that the vectors $\vec{v_1} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ and $\vec{v_2} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ span $\mathbb{R}^2$. Write $\vec{w} = \begin{bmatrix} -5 \\ 7 \end{bmatrix}$ as a linear combination of $\vec{v_1}$ and $\vec{v_2}$.

3. Can 2 vectors span $\mathbb{R}^3$?

4. Can 3 vectors be linearly independent in $\mathbb{R}^2$?